Math 261 — Exam 1

October 15, 2018

The use of notes and books is **NOT** allowed.

Exercise 1: Since today is October 15th... (30 pts)

In this exercise, you must justify the primality of any number larger than 50.

1. (8 pts) Find the factorization of 1510 into primes. Deduce the **number** of positive divisors of 1510.
2. (8 pts) Find the factorization of 1015 into primes. Deduce the **sum** of the postive divisors of 1015.
3. (14 pts) Compute $\phi(l)$, where $l = \text{lcm}(1510, 1015)$.

Exercise 2: Quotient=remainder (10 pts)

Find all positive integers $n \in \mathbb{N}$ such that in the Euclidean division of n by 261, the quotient is the same as the remainder.

Exercise 3: An lcm (10 points)

Let $n \in \mathbb{N}$. Determine $\text{lcm}(n, n+1)$ in terms of n.

PLEASE TURN OVER

Exercise 4: All in one (50 pts)

The purpose of the exercise is to find all integers $x, y \in \mathbb{Z}$ such that

$$\begin{cases} 21x + 30y = 6, \\ x \equiv 2 \pmod{7}, \\ y \equiv 1 \pmod{10} \end{cases} \qquad (\star)$$

This exercise is designed so you can explain how to solve a question even if you were unable howto solve the previous one.

1. (12 pts) First of all, let us focus on the equation $21x + 30y = 6$. Explain why there are numbers e, f, g, h such that the solutions are given by $x = e + ft$, $y = g + ht$ for $t \in \mathbb{Z}$, and find these numbers.

In the next questions, we are going to determine which $t \in \mathbb{Z}$ ensure that the other equations $x \equiv 2 \pmod{7}$ and $y \equiv 1 \pmod{10}$ are also satisfied.

2. (10 pts) We now plug in the condition $x \equiv 2 \pmod{7}$, that is to say $e + ft \equiv 2 \pmod{7}$ where e and f were found above. Explain why this is equivalent to $t \equiv k \pmod{7}$, where k is a constant that you must determine.

3. (5 pts) Similarly, show that the condition $g + ht \equiv 1 \bmod 10$ (where g and h were found in part 1.) is equivalent to the condition $t \equiv l \bmod 10$, where l is a constant that you must determine.

4. (18 pts) Explain why the two conditions

$$\begin{cases} t \equiv k \pmod{7}, \\ t \equiv l \pmod{10} \end{cases}$$

found in the previous two parts are equivalent to $t \equiv m \pmod{70}$ for some constant m, and find such an m.

5. (5 pts) Finally, what are the solutions to the system of equations $(\star)$?

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