ABSTRACT

This paper presents the controller for the Self-energising Electro-Hydraulic Brake (SEHB). A non-linear controller is chosen for the brake torque control of the railway brake, which has the major benefit that it achieves predictable dynamics of the brake torque build-up throughout the working range of the SEHB without controller adjustments. While being more complex in the implementation, the non-linear controller is more straightforward in the design when compared...
to a proportional controller, which needs to be tuned empirically. This paper focuses on the design of the controller, including the required simplification of the mathematical model to third order. Simulation results show that the desired dynamics can be achieved. Test rig measurements confirm the applicability of the non-linear brake torque controller.

**Keywords:** Hydraulic brake, railway brake, self-energising, brake torque control, input-output linearisation, Luenberger observer, state space controller

1 INTRODUCTION

Conventional railway brakes utilise pneumatics as an energy source for the braking process [1]. A centralised air compressor located in the locomotive generates the pneumatic energy. As every wagon is equipped with one or more pneumatic brakes, pneumatic pipes and hoses starting at the compressing unit run through the whole vehicle. The pneumatic lines through the train supply both energy and control signals to each brake. Due to the compressibility of the air and the resulting high capacitance of the fluid lines, the brake unit’s reaction times increase with their distance from the locomotive. This leads to an uneven brake torque distribution. The electro-pneumatic brake (EP-brake) overcomes this issue by transmitting the brake command electrically to the brake control units located in each wagon [2]. But still, pneumatic lines are necessary as a power supply. Their main advantage is a simple implementation of a fail-safe emergency and parking brake. New mechatronic brake systems have been developed with the aim to reduce complexity of the design interfaces between the subassemblies of a wagon which in part are caused by the pneumatic lines and their protection. These mechatronic brake designs commonly utilise an electrical command signal, examples of which are electro-mechanical brakes such as the electronic wedge brake (EWB) [3].

Since the early days of vehicle brake development, contributions were made to reduce the power consumption for brake actuation. To take advantage of the vehicle’s kinetic energy, self-reinforcing or self-energising principles can be applied. Innovations in the past made use of re-
duced actuation force through designs such as the self-reinforcing drum and disc brakes [4], [5], [6].

The challenge for electrified brakes usually is the high power demand for the short period when the clamping force has to be established and the high reliability demanded under harsh conditions. Hydraulic solutions offer a high power-density compared to electrical drives and are highly reliable due to their simplicity in design. At the RWTH Aachen University, Institute for Fluid Power Drives and Controls (IFAS) a novel self-energising hydraulic brake system has been developed [7], [8], [9], [10], [11], [12]. Besides being a closed, sealed system with only electric interfaces the brake system is self-energising, meaning that it draws its actuation energy almost exclusively from the vehicle’s kinetic energy. The system only requires energy on a low level for the actuation of control valves. As the SEHB is a novel railway brake concept, some changes of the infrastructure within the train set are necessary. However, these changes are minor and the existing brake control infrastructure, as currently present in trains with EP-brakes, can be used to actuate the SEHB. The topic of integration into the existing brake control infrastructure and compatibility with other brake systems is further elaborated on in [13]. Fig. 1 shows a schematic of the developed Self-energising Electro-Hydraulic Brake, representing only one of many possible implementations of the concept. Different implementations are presented and composed in [10] and [13].
The brake consists of the brake pads, a calliper, a brake actuator, a supporting cylinder and a control valve. Its functionality is best explained by considering a braking process: The piston side chamber of the brake actuator is pressurised, causing the brake actuator to move outwards and thereby clamping the brake pads onto the brake disc via the calliper. The radial guidance allows for a circular movement of the whole calliper, including the brake actuator and the brake pads. As can be seen in the lower view of Fig. 1, the supporting cylinder is mounted via a swivel to the calliper and its rod is connected to the vehicle chassis.

The circular movement of the brake calliper is continued by the supporting cylinder. As the calliper moves, oil in the supporting cylinder is compressed and pressure builds up. The piston area of the supporting cylinder is chosen in such a way to ensure that its pressure is always higher than the hydraulic pressure in the brake actuator by which it is caused. By opening the control valve in positive direction, pressurised oil flows from the supporting cylinder to the piston side chamber of the brake actuator, increasing the clamping pressure of the brake pads onto the disc. This leads to a higher supporting cylinder pressure which in turn increases the clamping pressure. This is the principle of self-energisation. To reduce the brake force and to retract the brake pads
off the brake disc, the valve opens negatively and connects the high pressure supporting cylinder chamber with the rod side chamber of the brake actuator.

In self-energisation the braking process is unstable if not controlled, which means that the brake will lock the wheel when the control valve position is maintained to allow pressure build-up. The self-energisation and its proportional control are explained analytically in [7]. However, the proportional controller for the SEHB has its challenges. Due to the wide range of brake force level range the gain of the proportional control has to be adjusted to the operating point. This requires extensive tuning. In the next section a method to control this instable process with a straight forward control strategy is presented based on input-output linearisation.

2 LINEARISATION AND CONTROLLER DESIGN

The mathematical equations describing the behaviour of the SEHB are non-linear, namely the orifice equation for the control valve flow. The non-linear control path can be linearised by non-linear feedback [8]. Fig. 2 illustrates the structure of the linearising feedback with the method of input-output linearisation. The input-output linearising feedback creates a new control input \( v \).

Assuming the successful implementation of the linearising feedback, the input/output dynamics \( \frac{v(s)}{y(s)} \) are linear.

![Fig. 2: Linearising Feedback with Coordinate Transformation](image)

A proportional state space control can be applied to the linearised system and the desired dynamics of the system can be tuned by pole placement.
The feedback linearisation requires the feedback of all the states. However, several state variables are not intended to be measured. Therefore a full state Luenberger observer is developed to estimate the unmeasured states (see Fig. 3). This strategy is well known and has been applied for other hydraulic control systems such as position control. [14] shows that for position control a proportional-integral state feedback is required in combination with the input-output linearisation. This is not equally valid for application on the SEHB, which is basically a pressure control application with unstable open-loop dynamics.

A mathematical model of the SEHB is explained in the next section.

2.1 Mathematical Model of the Brake System

The hydraulic and mechanical systems of the brake shown in Fig. 1 consist of a number of mechanical and hydraulic components. But several simplifications are proposed. The low pressure accumulator acts merely as a low pressure reservoir with a large capacity. It is assumed that its pressure \( p_R \) can be considered constant. The high pressure accumulator has a total capacity of 5 ml to be fully charged. Fully charged, it behaves like a constant hydraulic volume with a stiffness comparable to a pipe. For the following it is assumed that the high pressure accumulator is

![Fig. 3: Controller and Observer](image)
fully charged and therefore can be treated as a portion of the connecting pipe between supporting cylinder and valve. Fig. 4 then illustrates the simplified diagram of the brake model for increasing and decreasing brake torque. The simulation and experimental results presented in section 3 and 4.2 of this paper justify the simplified modelling approach.

The supporting cylinder is a plunger cylinder with area $A_{SC}$ and position $x_{SC}$. During brake torque build-up the rod side chamber of the brake actuator in Fig. 1 is connected to the low pressure line which is assumed to be at constant pressure $p_R$. Its influence can be represented as a constant force. The clamping force generated by the brake actuator is translated to the supporting cylinder through the friction contact and a mechanical connection, which is expressed in the transmission ratio $i$ and the friction coefficient $\mu$. The combined stiffness of the brake pads and the brake caliper is modelled as $c_{cal}$. The friction coefficient $\mu$ between brake pads and brake disc is assumed to be constant. This is the most debateable assumption made in the modelling and is further discussed in section 4.2 about experimental results. Finally, the spring of the brake actuator is considered in the model as a constant force.

The resulting third order model was published in [8] and is outlined in the following. A distinction is made between brake force increase (referred to as Case A, positive valve opening) and brake force decrease (referred to as Case B, negative valve opening). The approach of switching
between brake force increase and decrease was also shown for a fourth order model of the SEHB in [15]. The hydraulic valve flow is characterised by the valve flow coefficient $B$, the relative valve opening $x_V$, the supporting cylinder pressure $p_{SC}$, the brake actuator pressure $p_{BA}$, the reservoir pressure $p_R$, and the valve signal $u_V$:

$$Q = \begin{cases} B x_V \sqrt{(p_{SC} - p_{BA})}, & u_V \geq 0, \text{ Case A} \\ B x_V \sqrt{(p_{BA} - p_R)}, & u_V \geq 0, \text{ Case B} \end{cases}$$ (1)

The cylinder friction is included in the model as static friction. For the modelling of the sealing friction it is distinguished between Case A and Case B. Even though the friction model introduces a discontinuity in the overall model, the model is smooth for each case. Note that the friction coefficient $\mu$ between brake pad and brake disc should not be confused with the cylinder friction. For simplicity reasons, only the equations for brake torque build-up are written in detail. For brake torque build-up the control valve is opened positively. The equations for brake torque decrease are analogous.

The control valve is modelled as a second order lag element [16].

$$\dot{v}_V = \omega^2_v K_v u_V - 2 \omega_v D_v v_V - \omega^2_v x_V$$ (2)

$$\dot{x}_V = v_V$$ (3)

With $v_V$ valve spool velocity, $x_V$ valve spool position, $u_V$ valve input signal and $K_v$ valve gain factor. The dynamic performance is characterised by the undamped natural frequency $\omega_v$ and the valve damping coefficient $D_v$.

For brake torque build-up the chambers of both cylinders are connected hydraulically. Since they are also mechanically connected through the friction contact it is sufficient to represent both chambers by only one hydraulic capacity and one mass. The pressure build-up (Case A) is expressed in equation (4), where $Q(p_{BA})$ is a term that represents the net flow into the combined hydraulic capacity $C_{eq}$.

$$\dot{p}_{BA} = \frac{x_V Q(p_{BA})}{C_{eq}}$$ (4)
The square root expression \( Q(p_{BA}) \) is defined by equation (5).

\[
Q(p_{BA}) = B \sqrt{\frac{2\mu l(p_{BA} A_{BA} - F_{Fr,BA} + F_{Sp,BA})}{A_{SC}}} + \frac{F_{Fr,SC}}{A_{SC}} - p_{BA} \tag{5}
\]

The combined hydraulic capacity \( C_{eq} \) takes the hydraulic and mechanical stiffness of the brake mechanism into account can be written as:

\[
C_{eq} = C_H + \frac{A_{BA}^2}{c_{cal}} \tag{6}
\]

In equations (5) and (6) \( B \) is the valve flow coefficient, \( C_H \) the hydraulic system capacity, \( c_{cal} \) the combined stiffness of brake pads and brake calliper, \( A_{BA} \) the brake actuator piston area, \( A_{SC} \) the supporting cylinder piston area, \( \mu \) the friction coefficient between brake pad and brake disc, \( i \) the mechanical force transmission ratio between brake actuator and supporting cylinder, \( F_{Fr,BA} \) the brake actuator friction force, \( F_{Sp,BA} \) the brake actuator spring force, and \( F_{Fr,SC} \) the supporting cylinder friction force.

Equations (2), (3), and (4) constitute the non-linear state space description of third order of the simplified model illustrated in Fig. 4. It can be expressed as

\[
\dot{x} = f(x) + g(x)u_y
\]

\[
y = h(x)
\tag{7}
\]

This is written out for Case A as

\[
\begin{pmatrix}
\dot{p}_{BA} \\
\dot{v}_V \\
\dot{x}_V
\end{pmatrix} = 
\begin{pmatrix}
x_f Q(p_{BA}) \\
C_{eq} \\
-2\omega_V D_V v_V - \omega_V^2 x_V \\
0
\end{pmatrix} + 
\begin{pmatrix}
0 \\
\omega_V^2 K_V \\
0
\end{pmatrix} u_y
\tag{8}
\]

\[
y = 
\begin{pmatrix}
1 \\
0 \\
0
\end{pmatrix} p_{BA} = p_{BA}
\]

The system is linearised by a non-linear control feedback in the following.
2.2 Linearising Feedback for the SEHB

The SEHB state space system given in equation (7) is non-linear. The aim of the exact input-output linearisation described by Isidori [17] is to design a feedback such that the system behaves like a linear system. For linear systems the relative degree is defined as the pole-zero excess in the transfer function [18]. It is also a measure of how often the output can be differentiated until the input appears in the derivative [19]. The output of the brake system is \( y = h(x) \). The first derivative with respect to time is

\[
\dot{y} = \frac{dh}{dt} = \frac{\partial h}{\partial x} \frac{\partial x}{\partial t} = \frac{\partial h}{\partial x} (f(x) + g(x)u_v)
\]  

(9)

The expression

\[
\frac{\partial h}{\partial x} f(x)
\]  

(10)

is a derivative of the scalar function \( h(x) \) along the vector field \( f(x) \), also called a Lie-derivative [20]. It can be written in short form

\[
L_f h(x) = \frac{\partial h(x)}{\partial x} f(x)
\]  

(11)

Analogous the derivative of \( h(x) \) along the vector field \( g(x) \) can be written as

\[
L_g h(x) = \frac{\partial h(x)}{\partial x} g(x)
\]  

(12)

Repeated Lie-derivates are defined recursively

\[
L^k_f h(x) = \frac{\partial \{L^{k-1}_f h(x)\}}{\partial x} f(x)
\]  

(13)

With the notation of the Lie-derivatives the first derivative of the output function is written more compactly as

\[
\dot{y} = L_f h(x) + L_g h(x)u_v
\]  

(14)

For the SEHB the Lie-derivative \( L_g h(x) \) equals zero.
Then the first output derivative is

\[
\dot{y} = \frac{\partial h(x)}{\partial x} f(x) = \left( 1 \vline 0 \vline 0 \right) \begin{pmatrix}
\frac{x_v Q(p_{BA})}{C_{eq}} \\
\frac{Q(p_{BA})}{C_{eq}} \\
-2\omega_v D_v v_v - \omega_v^2 x_v
\end{pmatrix} = \frac{x_v Q(p_{BA})}{C_{eq}}
\]

(16)

The second output derivative yields

\[
\ddot{y} = \left. \frac{d}{dt} \left( \frac{\partial h(x)}{\partial x} \right) \right|_{L_f h(x)} = \frac{\partial}{\partial x} \left( \frac{\partial h(x)}{\partial x} \right) \left( f(x) + g(x)u_v \right) = L_f^2 h(x) + L_g L_f h(x) u_v
\]

(17)

with

\[
L_g L_f h(x) = \begin{pmatrix}
x_v B^2 \frac{\beta}{2Q(p_{BA})C_{eq}} & 0 & \frac{Q(p_{BA})}{C_{eq}} & 0 \\
0 & \frac{\omega_v^2 K_v}{v_v}
\end{pmatrix} = 0
\]

(18)

The factor \( \beta \) characterises the mechanical connection between the two cylinders and is defined as

\[
\beta = \frac{2i\mu A_{BA}}{A_{SC}} - 1
\]

(19)

Therefore, the second output derivative of equation (17) simplifies to

\[
\ddot{y} = L_f^2 h(x) = \begin{pmatrix}
x_v B^2 \frac{\beta}{2Q(p_{BA})C_{eq}} & 0 & \frac{Q(p_{BA})}{C_{eq}} & 0 \\
0 & \frac{\omega_v^2 K_v}{v_v}
\end{pmatrix} \begin{pmatrix}
x_v Q(p_{BA}) \\
-2\omega_v D_v v_v - \omega_v^2 x_v
\end{pmatrix}
\]

\[
= B^2 \frac{x_v^2}{2C_{eq}^2} \beta + \frac{v_v Q(p_{BA})}{C_{eq}}
\]

(20)

The third output derivative is determined in the same manner

\[
\dddot{y} = L_f^3 h(x) + L_g L_f^2 h(x) u_v
\]

(21)
Now, the Lie-derivative \( L_g L^2_j h(x) \) is non-zero and therefore the input \( u_V \) is part of the third output derivative. This implies that the relative degree of the SEHB system is three.

\[
L_g L^2_j h(x) = \left( \frac{v_V B^2 \beta}{2Q(p_{BA}) C_{eq}} \begin{bmatrix} Q(p_{BA}) \\ x_V B^2 \beta \\ 0 \end{bmatrix} \right) \begin{bmatrix} 0 \\ \omega_V^2 K_V \\ 0 \end{bmatrix} = \frac{Q(p_{BA})}{C_{eq}} \omega_V^2 K_V
\]  

(22)

\[
L_j^2 h(x) = \left( \frac{v_V B^2 \beta}{2Q(p_{BA}) C_{eq}} \begin{bmatrix} Q(p_{BA}) \\ x_V B^2 \beta \\ 0 \end{bmatrix} \right) \begin{bmatrix} x_V Q(p_{BA}) \\ -2\omega_V D_V v_V - \omega_V^3 x_V \end{bmatrix} + \frac{Q(p_{BA})}{C_{eq}} \omega_V^2 K_V u_V
\]  

(23)

Equation (21) is then

\[
\ddot{y} = \frac{x_V v_V B^2 \beta}{C_{eq}^2} + \frac{Q(p_{BA})}{C_{eq}} \left( -2\omega_V D_V v_V - \omega_V^3 x_V \right) + \frac{Q(p_{BA})}{C_{eq}} \omega_V^2 K_V u_V
\]  

(24)

The linear and controllable state space description is obtained through coordinate transformation. The new \( z \) coordinates are defined as the output’s derivatives.

\[
z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} y \\ L_j h(x) \\ L_j^2 h(x) \end{bmatrix} = \begin{bmatrix} \frac{p_{BA}}{C_{eq}} \\ \frac{x_V Q(p_{BA})}{C_{eq}} \\ \frac{B^2 x_V^3}{2C_{eq}^2} + \frac{v_V Q(p_{BA})}{C_{eq}} \end{bmatrix}
\]  

(25)

It follows that the differential equation of the transformed state space variable vector is

\[
\dot{z} = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} L_j h(x) \\ L^2_j h(x) \\ L_j h(x) + L_g L^2_j h(x) u_V \end{bmatrix} = \begin{bmatrix} z_2 \\ z_3 \\ L_j h(x) + L_g L^2_j h(x) u_V \end{bmatrix}
\]  

(26)

The derivative of the coordinate \( z_3 \) is non-linear. Defining a new system input \( v \), the control valve input \( u_V \) can be written as
\[
\mathbf{u}_V = \frac{-L_f h(x) + v}{L_g L_f h(x)} = \frac{-x_v v_p B^2 \beta}{C_{eq}^2} \frac{2\alpha_v D_v v_v - \omega_v^2 x_v}{C_{eq}} \frac{Q(p_{BA})}{C_{eq}} + v
\]

(27)

Then the differential equation of the transformed state space variable vector yields

\[
\dot{\mathbf{z}} = \left( \begin{array}{c}
\mathbf{z}_1 \\
\mathbf{z}_2 \\
\mathbf{z}_3 \\
\end{array} \right) = \left( \begin{array}{c}
\mathbf{z}_2 \\
\mathbf{z}_3 \\
\frac{L_f^3 h(x) + L_g L_f^2 h(x) - L_f h(x) + v}{L_g L_f^2 h(x)} \\
\end{array} \right)
\]

(28)

It is obvious that the chosen feedback makes the system linear.

\[
\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{b}_v = \left( \begin{array}{ccc}
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0 \\
\end{array} \right) \mathbf{z} + \left( \begin{array}{c}
0 \\
0 \\
1 \\
\end{array} \right) v
\]

\[
y = \mathbf{c}^T \mathbf{z} = (1 \ 0 \ 0) \mathbf{z} = (1 \ 0 \ 0) \left( \begin{array}{c}
\mathbf{z}_1 \\
\mathbf{z}_2 \\
\mathbf{z}_3 \\
\end{array} \right) = \mathbf{z}_1 = p_{BA}
\]

(29)

The linearising feedback in equation (27) is the expression used to calculate the control valve input \(u_V\) from the controller feedback \(v\). The valve input \(u_V\) therefore depends on the valve spool velocity, position and on the square root of the brake actuator pressure. As can be seen from Fig. 3, the linearising feedback uses the estimates of the physical states since the actual states are not measured. These estimates are obtained from an observer which is discussed in the next section.

**2.3 Controller and Full State Observer**

The main goal of the linearising feedback is to facilitate the linear control design in a straightforward way. In this paper a proportional state feedback control as shown in Fig. 3 is proposed. The only measured state is the supporting cylinder pressure \(p_{SC}\) which, according to the simplified model, is directly related to the brake actuator pressure \(p_{BA}\). The set values for valve spool velocity and valve spool position are zero. Hence, the reference input vector \(\mathbf{z}_{ref}\) is
The input \( v \) then becomes

\[
v = k^T e = k^T (\mathbf{z}_{\text{ref}} - \hat{\mathbf{z}}) = (k_1 \mid k_2 \mid k_3) \begin{pmatrix} p_{BA,\text{ref}} - \hat{z}_1 \\ 0 - \hat{z}_2 \\ 0 - \hat{z}_3 \end{pmatrix} = k_1 (p_{BA,\text{ref}} - \hat{z}_1) - k_2 \hat{z}_2 - k_3 \hat{z}_3
\]

(31)

In equation (31) \( \hat{\mathbf{z}} \) is the vector of the estimated state variables. The observer shown in Fig. 3 is crucial since the states \( z_2 \) and \( z_3 \) depend on the valve spool velocity and position according to equation (25). For the railway application it is not practical and too expensive to measure these variables. Thus, a full state Luenberger observer is used. The next section explains the strategy of placing the observer and state feedback poles.

### 2.4 Controller Dynamics

According to Lunze [18] the closed-loop system should have higher dynamics than the open-loop plant but the observer should be significantly faster than the closed-loop. The poles or eigenvalues of the linearised plant are in the origin by definition of the linearising feedback. They are placed to the left due to the state feedback, Fig. 5 illustrates this. The eigenvalues of the observer are placed considerably further left of the eigenvalues of the closed-loop system. Lunze [18] recommends a factor between two to six between the observer poles and the closed-loop poles.

![Fig. 5: Poles of the SEHB Controller and Observer](image-url)
It is easy to show that the eigenvalues of the linearised system are in the origin. The dynamic matrix $A$ determines the location of the open-loop poles and they are calculated by solving the characteristic equation (32).

$$\det(sI - A) = \det \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = s^3 = 0 \tag{32}$$

It is obvious that all three open-loop poles $s_1$, $s_2$, and $s_3$, are zero. The locations of the closed-loop poles are determined by the matrix $A_K$.

$$A_K = A - bk^T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -k_1 & -k_2 & -k_3 \end{bmatrix} \tag{33}$$

By assigning the closed-loop system poles $s_{Ci}$ and comparing the characteristic polynomial in the factored form

$$(s - s_{c1})(s - s_{c2})(s - s_{c3}) = 0 \tag{34}$$

with equation (32) the state feedback gains are obtained. For the pole assignment it is not a priority to make the brake as responsive as possible. Rather, a desirable dynamics is derived from specifications in railway standards. Railway standards such as [21] set limitations for the deceleration of the vehicle for the sake of safety and comfort. Since the vehicle deceleration is directly related to the brake torque, the location of the closed-loop poles can be determined from [21].

Fig. 6 is a step response of the closed-loop system with the three poles at $s_{c1} = 20$ l/s. The set point value is reached after 0.5 s, which is well above the lower limit specified in [21], with this setting. The brake is faster than required, which is necessary for wheel-slip control applications. Limiting the command signal’s slope during normal operation easily allows a smooth and jerk-free braking.
Fig. 6: Step Response of Closed-Loop System with Poles \( s_{ci} = 20 \, \text{1/s} \)

By comparison of coefficients of the characteristic equation with equation (34) the controller coefficients are

\[
\mathbf{k} = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} -s_{c1}s_{c2}s_{c3} \\ s_{c1}s_{c2} + s_{c2}s_{c3} + s_{c1}s_{c3} \\ -s_{c1} - s_{c2} - s_{c3} \end{bmatrix} = \begin{bmatrix} 8000 \\ -3s^3 \\ -1200s^2 \\ 60s \end{bmatrix}
\]

In a next step the observer’s poles are assigned. The differential equation of the observer is

\[
\dot{\mathbf{z}} = A\mathbf{z} + \mathbf{b}v + \mathbf{l}(\mathbf{y} - \hat{\mathbf{y}}) = \begin{bmatrix} \dot{z}_2 \\ \dot{z}_3 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ v \end{bmatrix} + \begin{bmatrix} l_1(z_1 - \hat{z}_1) \\ l_2(z_1 - \hat{z}_1) \\ l_3(z_1 - \hat{z}_1) \end{bmatrix}
\]

As mentioned before the observer poles are placed three times further left of the closed-loop poles. The three observer poles are placed at \( s_{oi} = 60 \, \text{1/s} \). The characteristic equation then is

\[
\det(s\mathbf{I} - \mathbf{A} + \mathbf{l} \mathbf{c}^T) = 0
\]

The observer feedback is determined through a comparison of coefficients

\[
\det(s\mathbf{I} - \mathbf{A} + \mathbf{l} \mathbf{c}^T) = s^3 + s^2l_1 + sl_2 + l_3 = 0
\]

to be

\[
\mathbf{l} = \begin{bmatrix} -s_{o1} - s_{o2} - s_{o3} \\ s_{o1}s_{o2} + s_{o2}s_{o3} + s_{o1}s_{o3} \\ -s_{o1}s_{o2}s_{o3} \end{bmatrix} = \begin{bmatrix} 180 \frac{1}{s^3} \\ 10800 \frac{1}{s^2} \\ 216000 \frac{1}{s^4} \end{bmatrix}
\]

With this the differential equation of the observer becomes:
\[
\dot{\mathbf{z}} = \begin{pmatrix}
\dot{z}_2 \\
\dot{z}_3 \\
\vdots \\
0
\end{pmatrix} + \begin{pmatrix}
0 \\
0 \\
\nu
\end{pmatrix} + \begin{pmatrix}
\frac{1}{s}(z_1 - \dot{z}_1) \\
10800 \frac{1}{s^2}(z_1 - \dot{z}_1) \\
216000 \frac{1}{s^3}(z_1 - \dot{z}_1)
\end{pmatrix}
\] (40)

It is clear that the observer has only one measured input \(z_1\) which is the brake actuator pressure \(p_{BA}\).

The proportional state space controller uses the estimated state variables to calculate the input \(\nu\) of the linearised open-loop control path, see equation (31). The valve signal \(u_V\) is calculated with the laws of the input-output linearising feedback given in equation (27), which also uses the estimated states as illustrated in Fig. 3. To be able to do this the estimated states in \(z\)-coordinates are transformed to the original \(x\)-coordinates. The transformation is obtained by solving equation (25) for the \(x\)-coordinates.

The next section demonstrates the verification of the non-linear controller using simulation.

### 3 MODEL VERIFICATION

The simplified mathematical model of the SEHB, given in equation (8) assumes that the control valve connects both cylinders hydraulically during brake operation and that the positions are mechanically connected through the friction contact. This leads to the result that only one hydraulic capacity is sufficient for modelling and consequently one equation describes the pressure build-up. As the control valve is modelled as a second order lag element the mathematical model is of third order. To verify that the model is suitable and positive valve opening leads to a brake force increase, negative valve opening to a brake force decrease, simulations are carried out using a model in compliance with the schematic shown in Fig. 4. The model and controller parameters are given in Table 1.
Table 1: Simulation Model and Controller Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
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<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
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<td>5409 N/mm</td>
<td>$l_2$</td>
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<td>$D_V$</td>
<td>1.5</td>
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<td>$i$</td>
<td>0.736</td>
<td>$l_3$</td>
<td>216000 1/s³</td>
<td>$F_{Fr,BA}$</td>
<td>1800 N</td>
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<td>$C_H$</td>
<td>$1.6 \cdot 10^{-13}$ m³/N</td>
<td>$s_{C1}=s_{C2}=s_{C3}$</td>
<td>-20 1/s</td>
<td>$F_{Fr,SC}$</td>
<td>175 N</td>
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<td>$s_{O1}=s_{O2}=s_{O3}$</td>
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<td>$F_{Sp,BA}$</td>
<td>2700 N</td>
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<td>$A_{BA}$</td>
<td>5027 mm²</td>
<td>$K_V$</td>
<td>1</td>
</tr>
<tr>
<td>$k_3$</td>
<td>60 1/s</td>
<td>$A_{SC}$</td>
<td>766 mm²</td>
<td>$\mu$</td>
<td>0.35</td>
</tr>
<tr>
<td>$l_1$</td>
<td>180 1/s</td>
<td>$B$</td>
<td>$1.12 \cdot 10^{-7}$ m²√m³/kg</td>
<td>$\omega_N$</td>
<td>2200 1/s</td>
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</table>

First, the open-loop behaviour is illustrated. The control valve signal $u_V$ is fed to the control valve as a step function. Fig. 7 shows the input signal and the corresponding dynamic response of all three variables of the state vector $x$. The brake actuator pressure $p_{BA}$ builds-up progressively in the open-loop as long as the control valve is kept open. As the brake actuator pressure is in direct relation with the brake force, the brake force also builds-up progressively. At each step in the input signal $u_V$ the valve spool position $x_V$ overshoots and quickly reaches its desired position.
Fig. 7: Dynamics of the Open-Loop Obtained with the Hydraulic System Model

In a next step the non-linear controller consisting of the input-output linearising feedback, the Luenberger observer and the proportional state space controller is applied. Additionally, an accumulator is added to the system. The only controller input is the brake force set point from which the brake actuator pressure is calculated. Fig. 8 shows step responses of a simulation using a detailed hydraulic system simulation as presented in [8] and composing it to a simulation where the open-loop is ideally linearised according to equation (29). The controller parameters for both simulations are the ones given in Table 1. The curve marked by squares represents the results which would be achieved if the system was ideally linearised and in absence of disturbances. It can be seen that a smooth brake force build-up is achieved and the controller stabilises the braking process effectively. The brake force build-up dynamics for a step from 4000 N to 6000 N is equal to the brake force build-up dynamics of the step from 3000 N to 4000 N as well as the brake release steps. This is the expected behaviour for a linear system. The non-ideal closed-loop behaviour is demonstrated by the cross-marked curve. It is evident that during the step from 3000 N to 4000 N the high pressure accumulator is being filled, which delays the brake force build-up. The brake force build-up from 4000 N to 6000 N is slightly slower than
expected from the linearised model, which could be due to an estimation error. Also, the dynamics of brake force increase and decrease are not identical as expected from the linear model. However, the influence is small and it is demonstrated that the brake dynamics can be largely set through a mathematically consistent control strategy without the necessity of tuning. This verifies the applicability of the non-linear controller for the SEHB.

As stated previously, the small discontinuity in the brake force during the step from 3000 N to 4000 N is due to the charging of the high pressure accumulator. Part of the oil delivered from the supporting cylinder flows to the accumulator during charging. The lower plot of Fig. 8 illustrates that the estimation errors $e_{s1} = p_{BA} - \hat{p}_{BA}$, $e_{s2} = x_{B} - \hat{x}_{B}$ and $e_{s3} = v_{B} - \hat{v}_{B}$ all converge. Finally it is concluded from the verification through simulation that the non-linear controller is able to provide brake force dynamics in compliance with standard BS EN 13452-1 [21].

Now, one might ask, what the contribution of the input-output linearisation is to the control performance. In Table 2 simulation results of step responses obtained with the detailed hydraulic system simulation model and different types of control are shown. The table compares the time
for brake force build-up for different brake force steps with the presented non-linear controller with results obtained with a specially empirically tuned proportional controller.

**Table 2: Simulation Step Response Results**

<table>
<thead>
<tr>
<th>Brake Force Steps</th>
<th>Ideal</th>
<th>Proportional Controller</th>
<th>Non-linear Controller</th>
</tr>
</thead>
<tbody>
<tr>
<td>3000 N to 4000 N</td>
<td>0.42 s</td>
<td>1.07 s</td>
<td>0.61 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>+155%</td>
<td>+45%</td>
</tr>
<tr>
<td>4000 N to 6000 N</td>
<td>0.46 s</td>
<td>0.43 s</td>
<td>0.70 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-7%</td>
<td>+52%</td>
</tr>
<tr>
<td>6000 N to 8000 N</td>
<td>0.46 s</td>
<td>0.92 s</td>
<td>0.47 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>+100%</td>
<td>+2%</td>
</tr>
<tr>
<td>8000 N to 10000 N</td>
<td>0.46 s</td>
<td>0.77 s</td>
<td>0.46 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>+67%</td>
<td>±0%</td>
</tr>
<tr>
<td>10000 N to 12000 N</td>
<td>0.46 s</td>
<td>0.66 s</td>
<td>0.44 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>+43%</td>
<td>-4%</td>
</tr>
<tr>
<td>12000 N to 14000 N</td>
<td>0.46 s</td>
<td>0.58 s</td>
<td>0.42 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>+26%</td>
<td>-9%</td>
</tr>
<tr>
<td>14000 N to 16000 N</td>
<td>0.46 s</td>
<td>0.52 s</td>
<td>0.41 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>+13%</td>
<td>-11%</td>
</tr>
<tr>
<td>16000 N to 18000 N</td>
<td>0.46 s</td>
<td>0.46 s</td>
<td>0.41 s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>±0%</td>
<td>-11%</td>
</tr>
</tbody>
</table>

When examining the step responses above 6000 N where the high pressure accumulator is already charged, it is clear that the proportional controller has a strong dependency on the operating point manifested in a decreasing rise time with increasing brake force level. In contrast, the non-linear controller adapts to the operating point as intended, and maintains a more constant step response. To achieve similar results with proportional control the gain has to be adapted to the brake force level, which is basically what the non-linear controller does. The advantage of the non-linear controller is that it was based on physical insight rather than on empirical tuning.

In a next step the non-linear controller is implemented in test rig measurements, which is presented in the next section.

## 4 TEST RIG MEASUREMENTS

Measurement results presented in this section verify the applicability of the proposed non-linear control. The match between simulation and measurement also justifies the system simulation as a valid tool to test future control developments.
4.1 Test Rig

The test bench consists of the SEHB, the driving unit and the signal processing equipment. A front view of the test rig is shown in Fig. 9. The original railway brake disc with a diameter of 640 mm is mounted on a shaft with two clamping sets. The shaft itself is fixed to the T-slot test bed with spherical roller bearings. Each end of the brake disc shaft is connected via a jaw-type clutch to a hydraulic axial piston motor. Those two driving axial piston motors have a volumetric displacement of 250 cm³ each and are supplied by a central pressure supply unit.

![SEHB Test Rig Diagram](image)

**Fig. 9: SEHB Test Rig**

The brake is mounted on a frame to the test bed. Accumulators and the control valve are mounted to the brake frame. Also shown in Fig. 9 are the brake calliper, the supporting cylinder and the amplification and signal conditioning equipment. A dSPACE real-time control board is used to process the test rig measurement signals and to send command signals to the control valve. A safety cage around the brake disc serves as protection. Further details on the SEHB test setup can be found in [13].
4.2 Measurement Results

The non-linear controller, including the valve command signal given in equation (27), is implemented in C-code in Matlab Simulink in an S-function. The control is executed by the dSPACE real-time hardware with a sampling frequency of 2.9 kHz. Fig. 10 shows the measurement results corresponding to the simulation experiment of Fig. 8.

![Fig. 10: Measurement Results with Non-Linear Controller](image)

A step response from 3000 N to 4000 N and from 4000 N to 6000 N is shown. Higher step responses could not be performed due to the power limitations of the test rig drive. When examining the brake force build-up at the step from 3000 N to 4000 N, a peak in the measured force at 1.6 s is visible. This peak occurs because the accumulator is charged and at 1.6 s the charging is finished and so the whole flow is suddenly available for the brake actuator. This can also be observed in simulation in Fig. 8. In the brake force measurement oscillations with a frequency of 26 Hz and lower frequency oscillations of around 1 Hz can be seen. The latter ones are probably caused by fluctuations of the friction coefficient $\mu$, which are not accounted for in the controller but influence the brake force estimation indirectly. Brake force oscillations are a common phe-
nomenon but often not recognised because they usually cannot be measured. Also, when several 
brakes are simultaneously acting, the brake force fluctuations are superposed and do not cause 
uncomfort unless they are audible.

After the brake force level is reached and the control scheme is switched, in the experimental 
results oscillations are present which are not evident in the simulation. There are several reasons 
for this. Besides dynamic brake force changes due to natural fluctuations of the brake force, the 
valve model is not very accurate around its zero position. An industrial servo valve with a nominal 
flow of 20 l/min is used for the experiments. As a flow of less than 1 l/min is necessary to 
build-up the brake force, the valve opening is below 5% and in this range of very small valve 
openings leakage effects become relevant. Therefore, small oscillations of the controller signal 
are directly visible in the brake force progression. Switching between the two controllers (Case 
A and Case B) after reaching the brake force level might also be a cause for the low frequency 
oscillations. To reduce the chattering, a small insensitivity range is included.

The rise times for the brake force decrease are in compliance with the results of the hydraulic 
simulation model. The lower plot of Fig. 10 shows the step from 4000 N to 6000 N in more de-
tail. The dynamics of measurement and simulation during the brake force build-up match very 
well. It is concluded that the detailed system simulation presented in [8] is suitable for the con-
trol design.

The described non-linear input-output linearising controller with full state observer is an appro-
priate approach for the self-energising brake system. It allows the control of the brake force with 
prescribed dynamics for different operation points without the necessity to empirically adjust a 
controller.

5 CONCLUSION

This paper presents the design and implementation of a non-linear controller based on in-
put/output linearisation for a self-energising electro-hydraulic brake for railway applications. The 
dynamic response of the closed-loop is tuned by the pole placement technique to fulfil railway
specifications and provide sufficiently high bandwidth for future implementations of wheel slip control. The controller is based on a non-linear simplified system model of third order. The control effectively reduces the dependency of brake force rise time on brake force level as compared to proportional control. In the range between 3000 N to 18000 N the brake force response time deviates up to 52% in case of the non-linear control as opposed to 155% in case of a proportional control.

Measurement results verify the simulation results and justify the applicability of the system simulation for use in future studies on control design. A drawback of the non-linear approach is that brake force and friction coefficient cannot be determined independently of each other without measuring both the supporting cylinder and the brake actuator load pressure. The controller assumes the friction coefficient to be constant which is not actually the case during operation. Future work will address this issue by using a second pressure measurement signal to implement a friction coefficient observer.

Electronic compensation of the non-linearities of the SEHB by input/output linearisation is only one option to achieve predictable and safe brake behaviour. The authors are currently working on the design and implementation of a hydro-mechanical flow control valve with electrically controllable flow set-point [22].

6 ACKNOWLEDGEMENTS

The authors express their thanks to the German Research Foundation (DFG) for funding the research on the SEHB. Further thanks go to Schaeffler Technologies GmbH & Co. KG for providing the authors with bearings for the test rig.

7 LITERATURE


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<tr>
<td>( \mathbf{c} )</td>
<td>Output Vector</td>
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<td>$\omega_v$</td>
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