1 INTRODUCTION

Broadly defined, revenue management (RM)\(^4\) is the process of maximizing revenue from a fixed amount of perishable inventory using “market segmentation” and “demand management” techniques. While RM is not new (in fact it is as old as commerce, e.g., haggling in a market can be considered a form of RM), the theory and practice of RM have seen significant scientific and practical advances in the last few decades, starting with the Airline Deregulation Act in 1978, which opened the door for RM in the airline industry. It is not surprising that airlines adopted RM, as most of the market characteristics conducive to RM are present. RM is considered an essential function of any airline due to the highly uncertain and competitive marketplace.

Consequently, airlines have some of the most sophisticated RM implementations around. As we discuss RM in more detail, we will illustrate concepts using examples (mainly) from the airline industry. We do this because of the importance of the airlines to the development of RM, and because air travel is common enough that most people have experienced airline RM (perhaps unknowingly). In addition, we have some industry background in airline RM. Despite this focus on the airline industry, we note that RM has expanded to many different industries, starting

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\(^4\) “Yield management” is another common terminology for RM. For details on terminology and the scope of RM, we refer the interested reader to Talluri and van Ryzin [16] and Weatherford and Bodily [18].
with industries that share similar characteristics with the airline industry, such as hotels and car rental agencies (Boyd and Bilegan [4]), and then to many other industries, including retailing and manufacturing industries; see Table 1 for a sample of industries that have implemented RM-based approaches.

RM applications have been highly successful, with benefits in the billions of dollars in some cases. For example, American Airlines reported an estimated $1.4 billion from applying RM techniques over three years in the early nineties (Smith et al. [14]), and has later reported an estimated annual benefit of $1 billion from implementing RM (Cook [8]). Boyd [5] estimates an increase in revenue in the order of 2-8% due to implementing RM in an airline. In the rental car business, National was able to escape liquidation and generate $56 million incremental revenue due to RM (Geraghty and Johnson [10]). Moreover, Hertz indicated that the implementation of an RM system yielded an increase in revenue in the order of 1-5% (Carrol and Grimes [7]). Other successful examples of RM implementation are abundant.

In the following, we first present the terminology that will be used throughout this chapter, and then discuss market segmentation and the other market characteristics often associated with RM.

1.1 TERMINOLOGY

Since we will illustrate RM concepts using airline examples, we first present some basic (airline) RM terminology that will be used throughout.

**Fare-class (class):** Each market segment is represented by a fare-class. We will index fare-classes such that a lower index refers to a higher-valued customer segment, i.e., fare-class 1 has the highest ticket price or fare of any class.

**Itinerary:** The set of specific flights a traveler uses to fly between his/her origin and destination.
Product: A combination of an itinerary and a fare-class.

Booking limit: The maximum number of tickets (seats) that can be sold to each fare-class for a particular flight.

Overbooking limit: The total number of tickets (seats) that can be sold for a particular flight; this limit is typically larger than the aircraft’s capacity in anticipation of travelers canceling their reservations or not showing up for their flights.

1.2 MARKET CHARACTERISTICS CONDUCIVE TO RM

As market segmentation is an essential part of RM, we will discuss it in some detail. Market segmentation depends on a heterogeneous customer base with diverse consumer preferences. The goal of market segmentation is to take what might seem an identical product or service, and somehow differentiate it from the consumers’ perspective. A good example is a coach seat on any flight. Despite the fact that the service the customer receives (i.e., flying with a coach seat) is nearly identical, there can be a great disparity on the price paid for a seat on a flight. This is because airlines try to segment the market into “business” and “leisure” customers, based on certain likely characteristics of each segment. Leisure customers usually book earlier, are more flexible concerning travel times, are more likely to stay at their destination over a weekend, are more certain of the trip and thus do not require refundable tickets, and are more price sensitive than business customers. Airlines therefore design their fare structures and booking rules (e.g., advanced purchase requirements, refundability, Saturday night stay) to segment the customer base, and thus charge business customers a premium (as they are usually less price sensitive), i.e., this is why a fully refundable ticket, bought six days before departure, for a trip without a Saturday night stay, is more expensive than a non-refundable ticket, bought a month before departure, with a Saturday night stay.
It is interesting to consider these segmentation rules. A fully refundable ticket is obviously a more expensive product for the airline to offer than a non-refundable ticket, as the airline faces the risk of an empty (and unpaid) seat if the customer decides to cancel the trip in the last minute. Likewise, it is only sensible for the airline to save a seat for a business passenger booking six days before departure if they pay more than the leisure passengers, as the airline risks not selling the seat. (This decision of how many products to reserve for the higher-valued classes, often termed “capacity control,” is where the fixed amount of perishable inventory comes into play.) In contrast, the Saturday night stay requirement is solely for segmentation purposes; it does not impact the airline in any other way. As can be seen, these “fences” (restrictions) are constructed so as to prevent customers of a high-valued class from “leaking” from their segment and buying at lower prices (although this remains possible).

Here, we will discuss, in more detail, the market characteristics that tend to favor the use of RM: 1) *Perishable inventory*. The products perish after a certain date. For example, an airline seat has no value after the flight departs; it cannot be “stored” for use later. A night-stay at a hotel must be used on the given night or, otherwise, the revenue opportunity from that room on that night will be gone. Other examples include seats for a sporting event, space on any means of transportation, electricity and other utilities, etc. (Weatherford and Bodily [17]). Obviously the concept of perishable inventory applies to service industries. What is not so obvious is that it may also apply to the manufacturing industry. Products themselves (e.g., cars, computers) perish after a certain date (last year’s computer might be nearly worthless now). Manufacturers producing customized products based on orders (i.e., on a “make-to-order” basis) do not typically carry finished-good inventories; hence, their production capacity is perishable. The concept of perishable inventory also applies to retailers selling, for example, fashion items,
seasonal items, or perishable grocery items. 2) **Fixed (limited) inventory.** Obviously, RM is relevant only if capacity is scarce with respect to demand. For example, an airline having airplanes large enough, to the extent that demand never exceeds capacity, need not worry about protecting seats for business travelers. 3) **Low marginal costs.** When accommodating an additional customer costs very little compared to the fixed cost of establishing the product, it becomes very important to sell to the highest possible number of customers (while, of course, satisfying the fixed capacity limit). For example, selling one more flight seat on a flight or one more room in a hotel will cost very little compared to other overhead costs. 4) **Demand uncertainty.** The limited ability to predict the future demand complicates demand management decisions (e.g., determining the appropriate “booking limit” for each fare-class). Most RM applications rely on probabilistic demand models that attempt to maximize the expected profit.

Individual RM implementations also depend on other market characteristics such as the consumers’ buying behavior, seasonality in demand, substitution/complementarity between the different products sold in the market, sales channels available to the firm, marketing and sales policies, relation of supply with respect to demand, and competition. As one might imagine, RM systems have become highly sophisticated, driven by intense competition, and enabled by scientific advances in the related disciplines as well as advances in information technology, which makes it possible to store, retrieve, and analyze vast amounts of data, and to implement complex algorithmic approaches to demand management decisions.

1.3 **OVERVIEW**

In this chapter, we present a representative cross section of RM models. Our objective is to give the reader a basic understanding of how RM effectively utilizes Operations Research (OR) techniques and methodology, while introducing the reader to the fundamentals of RM
methodology. Specifically, we focus on three areas of RM, which we believe are the most related to OR: Pricing, capacity control, and overbooking. Our presentation of pricing in Section 2 is cursory, and is mainly included to emphasize the benefits of price differentiation between customer segments. Our intention here is to illustrate how RM exploits a segmented market to maximize returns under fixed capacity by charging each customer “the right price,” which matches the customer’s willingness to pay. In Section 3, we discuss capacity control in some detail, as this is an area that has received the most attention in RM. In Section 4, we discuss the benefits, necessity, and practice of “overbooking.” Finally, in Section 5, we share our thoughts on the current and future challenges for RM.

2 PRICING

Consider an airline selling seats on a single flight to $n$ fare-classes. Let $C$ denote the capacity of the aircraft (i.e., the total number of seats available) and $p_i$ denote the ticket price for fare-class $i$, $i = 1, \ldots, n$, with $p_1 > p_2 > \ldots > p_n$. Assume that each fare-class is characterized by a deterministic demand function, $d_i(p_i)$, $i = 1, \ldots, n$. That is, if the price is set at $p_i$, then the demand for fare-class $i$ is $d_i(p_i)$. (Observe that the assumption that the demand functions are deterministic does not generally hold in practice, and is mainly made to simplify the problem and gain some insights.) With the ability to segment the market, and the existence of a fixed capacity and low marginal costs, as discussed in Section 1, the revenue management pricing problem with market segmentation under deterministic demand functions can be expressed as follows (see, for example, Bell [1]):

$$\max_{p_1, p_2, \ldots, p_n} \Pi^s = \sum_{i=1}^{n} p_i d_i(p_i)$$

subject to $\sum_{i=1}^{n} d_i(p_i) \leq C$. \hspace{1cm} (1)
The objective function in Model (1), denoted by $\Pi^S$, is the revenue generated from all fare-classes. The demand function, $d_i(p_i)$, is usually a decreasing function of $p_i$. Therefore, setting $p_i$ too low will produce a high demand, but might not maximize the revenue. On the other hand, setting $p_i$ too high would reduce demand, resulting in little or no revenue. The constraint in Model (1) reflects the fact that different fare-classes are competing for the limited capacity. This indicates that the airline is using prices to manage demand in order to match it with supply.

Suppose now that the airline is not willing or is unable to segment passengers into different fare-classes. In this case, the firm charges the same price, $p$, for all customers. The firm’s pricing problem without market segmentation then reduces to the following problem:

$$\max_{p} \Pi^{NS} = \sum_{i=1}^{n} pd_i(p)$$

subject to $\sum_{i=1}^{n} d_i(p) \leq C$.

It is easy to show that $\Pi^S \geq \Pi^{NS}$, that is, customer segmentation increases the airline’s revenue. Ignoring customer segmentation, as is done in Model (2), results in missed revenue opportunities. We illustrate this point further with an example.

**Example 1:** Suppose that Fly High Airlines (FHA) can segment the market for a particular flight into two distinct fare-classes (e.g., business vs. leisure), with demand curves given by $d_1(p_1) = 100 - 2p_1$ and $d_2(p_2) = 200 - 10p_2$, as illustrated in Figure 1. Suppose also that the aircraft assigned to this flight has a capacity of $C=150$ seats. Then, the pricing problem can be solved using Model (1), which has the following form:
Solving for the optimal prices under customer segmentation, we obtain \( p_1^* = \$25 \) and \( p_2^* = \$10 \).

The corresponding demands (i.e., number of seats sold to classes 1 and 2, respectively) are \( d_1(p_1^*) = 50 \) and \( d_2(p_2^*) = 100 \), with an optimal revenue (with segmentation) of $2,250. On the other hand, if the airline charges the same price for both fare-classes, then from Model (2), the optimal price will be the solution to:

\[
\max_{p} p(100 - 2p) + p(200 - 10p)
\]

subject to \( (100 - 2p) + (200 - 10p) \leq 150 \).

In this case the optimal price with no customer segmentation is \( p^* = \$12.5 \), with corresponding sales quantities \( d_1(p^*) = 75 \) and \( d_2(p^*) = 75 \). The optimal revenue without customer segmentation is $1,875. Thus, customer segmentation increases revenue from $1,875 to $2,250, by 20%! Figure 1 illustrates this revenue increase graphically, where the areas of the rectangles represent revenue, and the function, \( d(p) = d_1(p) + d_2(p) \), in Figure 1 (b) represents the total market demand without customer segmentation. □

The assumptions in Models (1) and (2) seldom apply in real life. First, demand is generally uncertain, time-dependent, and depends on the price(s) of all similar products sold by the firm as well as on other factors (such as competition, weather, special events) in a complex way. Firms engaged in RM use demand models that are far more sophisticated than these linear models. They often gather historical demand data and utilize sophisticated forecasting models to estimate the “form” (i.e., distribution and parameters) of the uncertain future demand.

Second, the ability to segment customers and determine the price-dependent demand function for each segment is not a straightforward task. As discussed in Section 1, firms need to
design their fare structures (i.e., construct fences) that prevent the high-valued customers (such as those of segment 1 in Example 1) from buying the products at prices set for the less-valued customers (such as those of segment 2 in Example 1, since \( p_2^* < p_1^* \)). Recall that in the airline industry, this is done by requiring the low-fare customers to book in advance, have a Saturday night stay, and pay high penalties in the events of cancellation or no-show. Nonetheless, leakage between the different segments remains possible, and further complicates the demand management problem.

As a result of the complexities in the demand and the business environment discussed above, most firms applying the RM methodology make their pricing and capacity decisions separately. In the remainder of this chapter, we will assume that prices have been determined, and study the problems of capacity control and overbooking.

3 CAPACITY CONTROL

In this section, we assume that the airline has determined the price for each fare-class, and is now attempting to maximize revenue by controlling the availability of its seats (which are perishable and limited in number). This involves determining whether or not to sell tickets for a certain fare-class at a given point in time (under the assumption of advanced purchase), or equivalently, determining how much inventory to reserve for each segment. As an example, consider an airline that offers two fare-classes on a given flight, with class 1 fare higher than class 2 fare as discussed above. Then the airline should never reject a class 1 customer as long as there is capacity available. Then the question that naturally arises is when to accept a class 2 customer. As discussed above, selling a seat to a class 2 customer runs the risk of not having a seat available for a higher-paying class 1 customer in the future (i.e., a business customer might
be “spilled”). On the other hand, rejecting a class 2 customer could result in the plane flying with empty “spoiled” seats. The capacity control decision revolves around this trade-off.

In Section 3.1, we discuss the single-resource (i.e., single flight) multi-class problem. Although many RM problems in reality involve networks, and hence require multiple resources (e.g., consider a customer who needs to take multiple flights between her origin and destination), it is not uncommon to solve such problems as a series of single-resource problems due to simplicity and flexibility. This, of course, translates into assuming that all resources are independent. In addition, these single-resource models provide basic insights into the aforementioned trade-off. Then, in Section 3.2 we study the multi-resource multi-class problem, also known as the “network revenue management,” “network capacity control,” or “origin/destination control” problem in the RM literature.

3.1 THE SINGLE-RESOURCE PROBLEM

Consider that the airline offers $n$ fare-classes with $p_1 > p_2 > \ldots > p_n$, and assume that class $n$ demand is realized first (i.e., class $n$ customers buy tickets first), followed by class $n-1$, then class $n-2$, etc., until class 1 demand is realized. This assumption is fairly realistic given the way the fare-classes are designed (see Section 2). Denote the demands for the different classes by independent random variables $X_i$, $i = 1, \ldots, n$. In addition, assume that there are no cancellations or overbooking. The problem is to determine how many ticket requests to accept for each fare-class.

The optimal solution to this problem can be obtained by dynamic programming (see, for example, Brumelle and McGill [6]). In particular, the structure of the optimal solution involves $n$ booking limits, $S_n, S_{n-1}, \ldots, S_1$, with $S_1 = C$, such that the airline accepts up to $S_i$ customers from class $i$, $i = 1, \ldots, n$, depending on the number of seats left after satisfying the demands for
classes \( n, n - 1, \ldots, i + 1 \) (each up to its own booking limit, of course). Observe that the booking limits result in a “nested” protection structure, \( y_1 \leq y_2 \leq \ldots \leq y_n \), with \( y_i, i = 1, \ldots n \), denoting the number of seats protected from (i.e., unavailable to) classes \( i \) to \( n \). Then \( y_i = C - S_i, i = 1, \ldots n \), see Figure 2.

There are certain drawbacks of the dynamic programming approach, however. As the number of fare-classes gets large (which is usually the case for most major airlines), so does the size of the resulting dynamic program, hence the computational times required to obtain the optimal booking limits. Consequently, in the following, we first present a special case with only two fare-classes (for this case, the optimal solution can be easily determined using the properties of the expected profit function), and then present a heuristic procedure for the general case having more than two classes.

### 3.1.1 The Single-Resource Two-Class Problem

We first consider a special case of the single-resource problem with two fare-classes only \((n = 2)\). The following model was first suggested by Littlewood [11], and is one of the earliest models for capacity control in RM. Recall that \( X_1 \) and \( X_2 \) respectively denote the demand for classes 1 and 2. \( X_1 \) and \( X_2 \) are both assumed to be non-negative, independent, and continuous random variables, with respective probability density functions \( f_{X_1}(.) \) and \( f_{X_2}(.) \).

As stated above, the form of the optimal policy is to sell \( S_2 \) seats to class 2 customers (under the assumption that the airline can sell as many class 2 tickets as it wants) and then “close” class 2 and accept only class 1 demand (up to capacity). Therefore, the airline’s expected profit for a given \( S_2 \) is

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\(^5\) This assumption is made to simplify the analysis; similar results can be obtained for the case where \( X_1 \) and \( X_2 \) are discrete random variables.
\[
E[\Pi(S_2)] = p_2 S_2 + p_1 E[\min(X_1, C - S_2)]
\]
\[
= p_2 S_2 + p_1 \left( \frac{C - S_2}{\int_0^{x_1} f_{X_1}(x_1)dx_1 + (C - S_2) \int_{C - S_2}^{\infty} f_{X_1}(x_1)dx_1} \right). \tag{3}
\]

The second term in the right hand side of (3) follows because the number of class 1 seats sold equals to \(C - S_2\) if \(X_1\) exceeds \(C - S_2\), and equals to \(X_1\), otherwise. Upon simplification, (3) reduces to
\[
E[\Pi(S_2)] = p_2 S_2 + p_1 \left( E[X_1] + \int_{C - S_2}^{\infty} (C - S_2 - x_1) f_{X_1}(x_1)dx_1 \right). \tag{4}
\]

Recall that the problem is to determine the optimal booking limit, \(S_2^*\), that maximizes the airline’s expected profit. It can be easily verified that function \(E[\Pi(S_2)]\) is strictly concave in \(S_2\). Hence, the optimal solution is unique, and the first-order optimality condition, given by
\[
\frac{\partial E[\Pi(S_2)]}{\partial S_2} \bigg|_{S_2 = S_2^*} = 0, \text{ is necessary and sufficient to determine the optimal solution, } S_2^*. \text{ Setting}
\]
\[
\frac{\partial E[\Pi(S_2)]}{\partial S_2} \bigg|_{S_2 = S_2^*} = 0 \text{ in (4) implies that } p_2 - p_1 \left( \int_{C - S_2^*}^{\infty} f_{X_1}(x_1)dx_1 \right) = 0, \text{ or equivalently,}
\]
\[
F_{X_1}(C - S_2^*) = \frac{p_1 - p_2}{p_1}, \tag{5}
\]
where \(F_{X_1}(\cdot)\) is the cumulative density function (CDF) of \(X_1\). Finally, rewriting (5) as
\[
p_2 = p_1 F_{X_1}(C - S_2^*), \tag{6}
\]
where \(F_{X_1}(x_1) = 1 - F_{X_1}(x_1) = P(X_1 > x_1)\), allows for another interesting interpretation of (5). The interpretation, which is due to Belobaba [2], is as follows: Accept a class 2 request as long as its price is greater than or equal to the expected marginal seat revenue (EMSR) of class 1, given by
\[
EMSR_1(C - S_2) = p_1 F_{X_1}(C - S_2). \tag{6}
\]
Note that (6) implies that \(p_2 > EMSR_1(C - S_2)\), for \(S_2 < S_2^*\),
see Figure 3 for a graphic illustration. (This interpretation is the basis for the heuristic for the single-resource multi-class problem discussed in Section 3.1.2.)

**Remark 1:** This problem is equivalent to a well-known inventory problem, the “newsvendor problem,” in which a newsvendor sells a daily newspaper. At the start of each day, the newsvendor must decide on the number of newspapers to purchase from the publisher at a price of $c$ per paper. Then during the day, she observes the random demand, $D$, which is modeled as a continuous\(^6\) random variable with CDF $F_D(.)$, and sells papers at a price of $r$ per paper. At the end of each day, the newsvendor can salvage any unsold newspapers for a price of $v$ per paper. The parameters are such that $r > c > v$ (otherwise, the problem becomes either trivial or ill-defined). Then it can be shown that the newsvendor’s optimal order quantity, $y^*$, satisfies

\[
F_D(y^*) = \frac{r - c}{r - v} = \frac{c_u}{c_o + c_u},
\]

where $c_u$ can be interpreted as the “underage cost,” i.e., the cost incurred per unit of unsatisfied demand, and $c_o$ can be interpreted as the “overage cost,” i.e., the cost incurred per unit of positive inventory remaining at the end of the period, with $c_u = r - c$ and $c_o = c - v$ (explain why). Then setting the “overage” cost, $c_o$, to $p_2$, and the “underage” cost, $c_u$, to $p_1 - p_2$ establishes the equivalence between the single-resource two-class problem discussed above and the newsvendor problem. □

We conclude this section with an example on the evaluation of $S_2^*$. 

**Example 2:** Consider again FHA, which has an aircraft with capacity $C = 150$ and offers two

\[^6\text{It is easy to extend the results to the case where the demand, } D, \text{ follows a discrete distribution.}\]
fare-classes at prices $p_1^* = \$25$ and $p_2^* = \$10$, as determined in Example 1. However, FHA has now better information about the demand, and has postulated that demand for class 1, $X_1$, can be modeled as a Normal random variable with mean $\mu_1 = 45$ and standard deviation $\sigma_1 = 20$. FHA must now decide on $S_2^*$, the optimal booking limit for class 2.

It follows from (5) that

$$S_2^* = C - (\mu_1 + Z \sigma_1),$$

where $Z = \Phi^{-1}\left(\frac{p_1^* - p_2^*}{p_1^*}\right)$ and $\Phi^{-1}(.)$ is the inverse of the standard Normal CDF. Therefore,

$$S_2^* \approx 150 - (45 + 0.253 \times 20) = 99.93.$$ Since in reality the airline will be restricted to discrete units, the airline should accept the first ninety-nine requests (since EMSR$_1(C-100) > EMSR_1(C-99.93) = p_2$, see Figure 3) from class 2 customers and reject the rest. In other words, the airline should protect $C - S_2^* = 51$ units for class 1 customers; see Figure 3 for a graphical illustration of this solution.

3.1.2 The Single-Resource Multi-Class Problem

We now revisit the single-resource problem with $n$ fare-classes. As mentioned above, the structure of the optimal solution involves $n$ booking limits, $S_n, S_{n-1}, \ldots, S_1$, with $S_1 = C$, which can be determined exactly by dynamic programming. However, as the number of fare-classes gets large, the computational times required to obtain the optimal booking limits with dynamic programming increase significantly. Therefore, in the following we will present an efficient heuristic, termed EMSR-B (see Belobaba [3]), which is widely used in practice. This heuristic generalizes the EMSR rule proposed in (6). In particular, the booking limit for class $i = 2, \ldots, n$ (with $S_i = C$) is given by

$$p_i = \bar{p}_{i-1} E_{W_i} (C - S_i^*),$$

(8)
where $\bar{p}_{i-1}$ is the “average” fare for classes $1, \ldots, i-1$, and $W_{i-1}$ is the sum of the demands of classes $1, \ldots, i-1$, that is,

$$
\bar{p}_{i-1} = \frac{i-1}{\sum_{j=1}^{i-1} E[X_j]} \sum_{j=1}^{i-1} p_j E[X_j], \quad W_{i-1} = \sum_{j=1}^{i-1} X_j, \text{ and } F_{W_{i-1}}(w) = P(W_{i-1} > w).
$$

(Compare (8) with (6).) It is commonly assumed that $X_i, i=1, \ldots, n$, is a Normal random variable with mean $\mu_i$ and standard deviation $\sigma_i$. In this case, $W_{i-1}$ is also Normal with mean $\sum_{j=1}^{i-1} \mu_j$ and standard deviation $\sqrt{\sum_{j=1}^{i-1} \sigma_j^2}$. Then, $S_i^*$ can be evaluated easily using a formula similar to (7) in Example 2.

### 3.2 THE MULTI-RESOURCE (NETWORK) PROBLEM

Not surprisingly, when several products share two or more resources, the RM capacity control problem becomes more complicated. For example, if an airline offers trips from city $A$ to city $B$ and from city $B$ to city $C$, then “connecting” passengers going from $A$ to $C$ through $B$ are a possibility, with $B$ acting as a “hub” (examples of this problem in other industries include multi-night stays at hotels or multi-day car rentals). This complicates the capacity control problem. We must now consider the question of how to value a connecting passenger. A connecting passenger might have a relatively high fare; so is accepting a connecting passenger on the flight from $A$ to $B$ using the rules presented in Section 3.1 based on this relatively high fare a good decision? What factors should be considered? Now imagine an airline with multiple hubs and thousands of flights a day. Clearly this is a difficult problem, which does not lend itself to an optimal solution with any reasonable assumptions. As such, in the following, we will present
two commonly used heuristic approaches for this problem: “bid price control” and “displacement-adjusted virtual nesting.” In fact, both heuristics have many variations in practice. Here we only describe basic versions of each.

3.2.1 The Bid Price Control Heuristic

The first approach models this capacity control problem on the origin/destination (OD) network (with multiple, dependent resources and multiple fare-classes) as a network flow maximization problem using expected demands from each product offered by the airline (see, for example, Boyd and Bilegan [4]):

\[
\begin{align*}
\max_{x_i, i \in I} & \sum_{i \in I} p_i x_i \\
\text{subject to} & \sum_{i \in I(l)} x_i \leq C_l, \ l \in L, \\
& x_i \leq E[X_i], \ i \in I, \\
& x_i \geq 0, \ i \in I,
\end{align*}
\]  

(9)

where \(I\) is the set of products offered, \(L\) is the set of flight legs (resources) in the network, \(I(l)\) is the set of products utilizing leg \(l\), \(l \in L\), \(C_l\) is the available capacity of leg \(l\), \(l \in L\), \(E[X_i]\) is the expected demand for product \(i\), \(i \in I\), and \(p_i\) is the price of product \(i\), \(i \in I\). The decision variables, \(x_i\), represent the number of reservations accepted for product \(i\), \(i \in I\).

Instead of utilizing the primal formulation given in Model (9) (and the corresponding decision variables, \(x_i, i \in I\)), a common approach is to solve the corresponding dual problem and obtain \(\lambda_l, i \in I\), the dual variable corresponding to the capacity constraint on leg \(l\), \(l \in L\). The dual variable \(\lambda_l, l \in L\), represents the “displacement cost” of accepting a passenger on leg \(l\), or equivalently, the minimum acceptable “bid price” for leg \(l\). Then, product \(i\) is made available for sale if its fare is greater than or equal to the sum of the bid prices for the legs it utilizes. That is, the “bid price control” policy is to accept a request for product \(i\) if

\[p_i \geq \sum_{l \in L(i)} \lambda_l,\]  

where \(L(i)\) is the
set of flight legs in product $i$ (see, for example, Boyd and Bilegan [4] and Talluri and van Ryzin [15]). In practice, the expected demand estimates, $E[X_i]$, and the available capacities, $C_i$, are frequently updated as the departure time approaches and more demand information is obtained, and new values of $\lambda_l$ are obtained (and hence, a new control policy leading possibly to closing some low-price fares is developed).

3.2.2 The Displacement-Adjusted Virtual Nesting (DAVN) Heuristic

DAVN is another commonly used heuristic for network capacity control (see, for example, Talluri and van Ryzin [15]). The idea is to determine a displacement cost for each leg (using, for example, the formulation in Model (9)), and then to decide on the capacity control of each leg separately utilizing the single resource methods discussed in Section 3.1. In particular, given displacement costs, $\lambda_l$, $l \in L$, we first calculate a “displacement-adjusted revenue,” $p_{il}$, for each product $i \in I$ and each leg $l \in L$, $p_{il} = p_i - \sum_{k \in L(\{i\} \setminus \{l\})} \lambda_k$, which approximates the net revenue for accepting product $i$ on leg $l$. Given the large number of products that use a given leg, a common approach is to cluster products into “virtual buckets” based on their displacement-adjusted revenues (this is also known as “virtual nesting”). Each virtual bucket is then treated as a separate product and booking limits are obtained for each bucket on each leg.

Under virtual nesting, a request for a product will be rejected if it falls in a bucket that has received reservations that exceed its booking limit on any of the leg that the product utilizes. Consider again the airline example above, and suppose that the product with origin $A$, destination $C$, and class 2 has been assigned to bucket 7 on leg $AB$ and to bucket 4 on leg $BC$. Then, a request for this itinerary will be rejected if either bucket 7 on leg $AB$ or bucket 4 on leg $BC$ has exceeded its booking limit.
Example 3: FHA operates between three cities on the East Coast, Boston (BOS), New York (JFK), and Washington DC (IAD) (see Figure 4). FHA utilizes IAD as a hub and flies one round-trip daily between IAD and each of the other two cities; see Table 2 for the flight information and capacities of the aircraft assigned to the flights. FHA offers tickets in two fare-classes, 1 and 2. As a result, it offers twelve products (i.e., itineraries for six OD pairs, each offered in two fare-classes; see the first column in Table 3, where each product is denoted by indices $ij$, with $i$ denoting the OD pair, and $j$ denoting the fare-class).

It is February 1 and FHA is determining its capacity control policy for flights on February 15. As of February 1, no bookings have been received for flights on February 15. The demand forecast for each product is broken into two periods (i.e., weeks), with period 1 preceding period 2. The fares and the period demand forecasts for each of FHA’s twelve products are given in Table 3, where $N(\mu, \sigma)$ denotes a Normal random variable with mean $\mu$ and standard deviation $\sigma$. FHA uses a bid price control policy as described in Model (9). Thus FHA’s network capacity control problem on February 1 can be formulated as follows:

$$\begin{align*}
\text{max} & \quad 203x_{11} + 63x_{12} + 303x_{21} + 93x_{22} + 204x_{31} + 44x_{32} + 304x_{41} + 94x_{42} + 203x_{51} + 53x_{52} \\
& \quad + 204x_{61} + 64x_{62} \\
\text{subject to} & \quad x_{11} + x_{12} + x_{21} + x_{22} \leq 70 \quad (\lambda_{\text{JFK-IAD}}) \\
& \quad x_{31} + x_{32} + x_{41} + x_{42} \leq 50 \quad (\lambda_{\text{BOS-IAD}}) \\
& \quad x_{41} + x_{42} + x_{61} + x_{62} \leq 70 \quad (\lambda_{\text{IAD-JFK}}) \\
& \quad x_{51} + x_{52} + x_{51} + x_{52} \leq 50 \quad (\lambda_{\text{IAD-BOS}}) \\
& \quad x_{11} \leq 13; \quad x_{12} \leq 39 \\
& \quad x_{21} \leq 11; \quad x_{22} \leq 28 \\
& \quad x_{31} \leq 14; \quad x_{32} \leq 40 \\
& \quad x_{41} \leq 12; \quad x_{42} \leq 31 \\
& \quad x_{51} \leq 12; \quad x_{52} \leq 39 \\
& \quad x_{61} \leq 11; \quad x_{62} \leq 38 \\
& \quad x_{ij} \geq 0, \quad i = 1, \ldots, 6, \quad j = 1, 2.
\end{align*}$$

(10)
Solving Model (10) (which can be done using an optimization software such as AMPL, see http://www.ampl.com) gives:

\[
\begin{align*}
\lambda_{\text{JFK} \rightarrow \text{IAD}} &= $40 \\
\lambda_{\text{BOS} \rightarrow \text{IAD}} &= $30 \\
\lambda_{\text{IAD} \rightarrow \text{JFK}} &= $64 \\
\lambda_{\text{IAD} \rightarrow \text{BOS}} &= $53
\end{align*}
\]

Consequently, under the bid price control policy, FHA would accept reservations for a product whose fare is greater than or equal to the sum of the bid prices for the flights it utilizes. Then, the bid price control policy for FHA on February 1 is to accept reservations for all products for February 15, see Table 4.

### 4 OVERBOOKING

Airline RM systems are based on advance reservations for a future travel itinerary. In many cases, customers have the right to cancel their reservation with little or no penalty. In other instances, customers may simply not show up for a flight (e.g. due to the vagaries of airline RM, one-way tickets are often not discounted, and thus are more expensive than round-trip tickets. This makes it cheaper for passengers to buy the round-trip ticket and not show up for the return flight. Can you guess why one-way tickets would be more expensive?). This can be a significant source of lost revenue. In fact, recent studies in the airline and rental car industries (Smith et al. [14] and Geraghty and Johnson [10]) report that on average only 50% of all reservations “survive” (i.e., the customer actually uses the product). To avoid this revenue loss, airlines commonly allow reservations to exceed capacity in anticipation that some reservations will not survive. This business practice is known as “overbooking.” Obviously, the drawback of overbooking is that it can lead to more products sold than capacity, hence some customers
being denied service. Therefore, it is important to set an “overbooking limit” appropriately in order to utilize *most* of the available capacity while honoring the reservations of *most* of the customers. RM focuses on setting the overbooking limits so as to maximize revenue while considering such “service level” constraints.

Historically, overbooking has its roots in the airline industry. However, it dates back to the sixties and seventies, prior to the deregulation of airlines and the subsequent development of modern RM (see Rothstein [12] for an extensive historical exposure). In the sixties and seventies, airlines used to engage in overbooking in a discrete manner without informing the customers of its consequences. A law suite won by Ralph Nader in 1976 changed this practice. The airlines became obliged to inform customers about overbooking (which they still do on the back of each ticket). Airlines also started developing innovative ways to make service denials more acceptable to customers. Motivated by research in economics (e.g., Simon [13]), some airlines currently manage overbooking as an auction. They offer compensation (such as a travel voucher of some monetary value, to be used for future travel) to get volunteers for service denials on flights for which more travelers than seats show up at the time of departure.

In the remainder of this section we present two simple, static (i.e., they ignore the dynamics of cancellations and new reservations over time) models for determining the overbooking limit to introduce the reader to some overbooking concepts. As always, in practice overbooking models are more sophisticated and are usually integrated with the other models discussed here.

4.1 DISTRIBUTION OF SHOWS (SURVIVALS)

Suppose it is estimated that a passenger will “show up” for the flight with probability \( q \) independently of the other passengers, that is, \( q \) is the probability that a reservation will “survive.” Suppose also that \( y \) customers have reservations at a given time. Then, out of the \( y \)
reservations, the probability that \( z \) reservations survive is

\[
P(Z(y) = z) = \binom{y}{z} q^z (1 - q)^{y-z},
\]

(11)

where \( Z(y) \) is the random variable representing the number of surviving reservations. This is known as the “binomial model” because \( Z(y) \) follows a Binomial distribution. This model is attributed to Thompson [16]. Although this model is based on several simplifying assumptions (e.g., it is static, it ignores people traveling in groups, who need to cancel their reservations together), it is desirable due to its simplicity. In the following, we present two methods to determine the overbooking limit: based on service level and expected profit.

4.1.1 Overbooking Limit Based on Service Level

Using the binomial model, we can determine the corresponding service level. Suppose that the overbooking limit is \( L \) (\( L > C \)). There are two commonly used service levels (see, for instance, [15]).

1. Type 1 service level: the probability that at least one customer will be denied service, that is

\[
s_1(L) = P(Z(L) > C) = \sum_{k=C+1}^{L} \binom{L}{k} q^k (1 - q)^{L-k}.
\]

(12)

2. Type 2 service level: the fraction of customers who are denied service, that is

\[
s_2(L) = \frac{E[(Z(L)-C)^+]}{E[Z(L)]} = \sum_{k=C+1}^{L} \binom{L}{k} q^k (1 - q)^{L-k} \frac{(k-C)}{Lq},
\]

(13)

where \( x^+ = \max(0, x) \).

A firm will set a desired service level (e.g., the probability that at least one customer is denied service is less than 1%, the percentage of customers who are denied service is less than 2%). The corresponding overbooking limit can then be calculated by solving for \( L \) in (12) or (13).
4.1.2 Overbooking Limit Based on Expected Profit

Suppose that each customer denied service incurs a cost $G$. For example, in the airlines, $G$ is the cost of a full refund and an additional reward ticket. Let $p$ be the price of the product. Then, the airline’s expected profit given that the airline sells $L$ tickets, where $L$ is the overbooking limit, is given by

\[ pL - G \sum_{k=C+1}^{L} \binom{L}{k} q^k (1-q)^{L-k}. \]  \hspace{1cm} (14)

Then, it can be shown that the optimal booking limit, $L^*$, is the largest value of $L$ that satisfies

\[ qP(Z(L-1) \geq C)G \leq p, \]  \hspace{1cm} (15)

or equivalently,

\[ qG \sum_{k=C}^{L-1} \binom{L-1}{k} q^k (1-q)^{L-1-k} \leq p. \]  \hspace{1cm} (16)

The left-hand side of (15) reflects the fact that in order for the $L^{th}$ reserving customer to be denied service, (i) there should be enough survivals from the first $(L-1)$ reservations to utilize all the capacity, and (ii) the $L^{th}$ customer should survive (with probability $q$).

**Remark 2.** When dealing with several customer segments that will show up for service with different probabilities, a common approach is to approximate the survival probability, $q$, by a weighted average of the survival probabilities of the segments (Talluri and van Ryzin [15]).

**Example 4.** FHA actually flies a 43-seat Embraer RJ145 between BOS and IAD. FHA

---

7 This result follows because the expected profit function in (14) is concave in $L$. 
estimates that the survival probability for this leg is 0.86. The overbooking limit of 50 used in Example 3 was obtained utilizing a simple heuristic, which used the ratio of the actual capacity to the survival probability (i.e., $43/0.86 = 50$). FHA now wants to use more sophisticated techniques so as to obtain a “better” overbooking limit. FHA is evaluating two alternatives:

(i) Setting the overbooking limit in a way that the percentage of customers denied boarding is less than 1%.

(ii) Setting the overbooking limit in a way that maximizes the expected profit. FHA estimates that a customer denied boarding costs $500 and that the average fare is $85 (this is approximately the weighted average of class 1 and class 2 fares in Table 3 with the weights being the mean demands for the two classes).

The booking limit required in (i) can be obtained from (13) as the largest value of $L$ that satisfies

$$\sum_{k=44}^{L} \binom{L}{k} (0.86)^k (0.14)^{L-k} \leq 0.01.\,$$

Searching over $L = 44, 45, \ldots$, it can be seen that $L^* = 48$ is the appropriate overbooking limit with a percentage of customers denied boarding of 0.7%.

The booking limit required in (ii) can be obtained from (16) as the largest value of $L$ that satisfies

$$430 \sum_{k=43}^{L-1} \binom{L-1}{k} (0.86)^k (0.14)^{L-1-k} \leq 85.\,$$

Searching over $L = 44, 45, \ldots$, it can be seen that $L^* = 48$ is again the appropriate overbooking limit. In conclusion, it seems that a booking limit of 50 is a bit high given the survival probability of 0.86. □
5 A CASE STUDY

You are the Manager of Revenue Optimization at FHA and your job is to improve the RM system (see Example 3 for the current system). FHA’s current system calculates the bid-prices for each departure date only once, at the beginning of the first period (see Example 3). The CEO of FHA has taken a class in RM, and suggests the following options to improve revenue:

(1) Upgrade FHA’s bid-price system so that it produces updated bid-prices at the beginning of period 2.

(2) Ignore the network effects, and simply use a flight-based (instead of a network-based) RM system (see Section 3.1). For products that consist of multiple flights, the stated fare will be used when determining the set of booking limits on each flight.

(3) Modify FHA’s bid-price system so that the optimal number of tickets (from Model (9)) for each product is used to limit ticket sales, i.e., if $x_{11} = 10$, then FHA will only sell up to 10 tickets for product 11. This will no longer be a bid-price system, but it is still based on Model (9).

(4) Use a DAVN system (see Section 3.2.2).

Of course, these four options are mutually exclusive. It is up to you to decide which option is best. (Alternatively you can come up with another option.) To present your solution to the upper management, you need to prepare a detailed report. You should support your decision with a detailed quantitative analysis. In addition, you should include a discussion on the following points in your report: What are the drawbacks and advantages of each option? Can you think of a simple way to improve on any of the options? Is each option expected to perform better: 1) under high or low demand uncertainty? 2) with high or low mean demands compared to capacity?
6 CHALLENGES AND FUTURE RESEARCH DIRECTIONS

In this chapter we present a brief overview of revenue management (RM), specifically focusing on the use of OR techniques in this field. We believe that this chapter will provide the reader with a basic understanding of RM and serve as a good starting point for those new to RM. We refer the reader interested in a more detailed and a comprehensive material to the excellent text by Talluri and van Ryzin [15] that we have consulted while preparing this chapter. The review article by Boyd and Bilegan [4] is another highly useful reference.

Finally, we point out some of the current challenges and future directions of RM that we believe are the most important. We believe that a major challenge for RM is in applications in areas beyond the travel industry (e.g., airlines, hotels, and rental cars). Boyd and Bilegan [4] identify the broadcasting industry and hospitals as two important areas for “non-traditional” RM applications. RM applications to trucking and manufacturing industries seem to be also promising.

Another challenge for RM is in coping with a changing, and a more competitive and uncertain, business environment. For example, in competing with low-cost carriers, major (legacy) airlines are bringing their fares down to low levels, which is jeopardizing profitability. This is prompting major airlines to come up with innovative techniques to benefit from their large fleets and networks. Gallego and Phillips [9] discuss such a novel approach. In particular, they consider a major airline flying multiple trips between two cities. In addition to the flight-specific products, the airline offers a cheaper, “flexible product,” which guarantees the customer a flight between the two cities on a certain date and within a certain time window, but without specifying the exact time of departure. This provides the airline with some flexibility, and allows it to hedge against demand uncertainty by allocating the flexible product customers to the
flights at a later time, when more demand information is obtained and uncertainty is reduced. Thus, the airline can better match its supply (capacity) with demand. This is just one example. We expect that such novel approaches that provide the firms with more flexibility, applied in conjunction with sophisticated RM techniques, will be the future of RM implementations.

RM is a discipline that is spreading to more and more industries, each with its own challenges. When a firm embraces RM, it is usually a core function of the firm, which impacts many other units such as marketing, sales, pricing, and scheduling. As such, RM needs to be in tune with the market, industry, and the firm.

**PRACTICE QUESTIONS**

1. What strategies can the manufacturing industry use to segment the market? Consider different types of manufacturing industries and discuss this question in the context of each industry. What types of manufacturing industries could benefit most from RM? Why?

2. What demand management decisions do retailers need to make? Answer this question in the context of different types of retailers.

3. How does the use of the internet facilitate RM implementation?

4. Explain why the equivalence between the newsvendor solution and that for the single-resource two-class problem discussed in Section 3.1.1 holds (see Remark 1).

5. Show that the expected profit function for the single-resource two-class problem, given in (4), is strictly concave in $S_2$.

6. Show that the expected profit function with overbooking, given in (14), is concave in $L$. Using this result, derive the optimality conditions in (15) and (16).
References


### Table 1 Who uses RM?

<table>
<thead>
<tr>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airlines (All)</td>
</tr>
<tr>
<td>Hotels (Hyatt, Mariott, Hilton, Sheraton, Forte, Disney)</td>
</tr>
<tr>
<td>Vacation (Club Med, Princess Cruises, Norwegian)</td>
</tr>
<tr>
<td>Car Rental (National, Hertz, Avis, Europcar)</td>
</tr>
<tr>
<td>Washington Opera</td>
</tr>
<tr>
<td>Freight (Sea-Land, Yellow Freight, Cons, Freightways)</td>
</tr>
<tr>
<td>Television Ads (CBS, ABC, NBC, TVNZ, Aus7)</td>
</tr>
<tr>
<td>UPS, SNCF</td>
</tr>
<tr>
<td>Retail (Retek, Khimetrics)</td>
</tr>
<tr>
<td>Real Estate (Archtone)</td>
</tr>
<tr>
<td>Natural Gas</td>
</tr>
<tr>
<td>Texas Children’s Hospital</td>
</tr>
</tbody>
</table>

Table 2 FHA’s flight schedule and capacity assignment

<table>
<thead>
<tr>
<th>Flight</th>
<th>Origin</th>
<th>Destination</th>
<th>Departs</th>
<th>Arrives</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>JFK</td>
<td>IAD</td>
<td>8:00 AM</td>
<td>9:30 AM</td>
<td>70</td>
</tr>
<tr>
<td>250</td>
<td>IAD</td>
<td>JFK</td>
<td>11:00 AM</td>
<td>12:30 PM</td>
<td>70</td>
</tr>
<tr>
<td>300</td>
<td>BOS</td>
<td>IAD</td>
<td>8:00 AM</td>
<td>9:30 AM</td>
<td>50</td>
</tr>
<tr>
<td>350</td>
<td>IAD</td>
<td>BOS</td>
<td>11:00 AM</td>
<td>12:30 PM</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 3 FHA’s demand forecasts and fares

<table>
<thead>
<tr>
<th>Product</th>
<th>Orig.</th>
<th>Dest.</th>
<th>Demand - Class 1</th>
<th>Demand - Class 2</th>
<th>Fares</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Period 1</td>
<td>Period 2</td>
<td>Period 1</td>
</tr>
<tr>
<td>11, 12</td>
<td>JFK</td>
<td>IAD</td>
<td>N(4, 1)</td>
<td>N(9, 3)</td>
<td>N(30, 7)</td>
</tr>
<tr>
<td>21, 22</td>
<td>JFK</td>
<td>BOS</td>
<td>N(3, 1)</td>
<td>N(8, 2)</td>
<td>N(20, 4)</td>
</tr>
<tr>
<td>31, 32</td>
<td>BOS</td>
<td>IAD</td>
<td>N(4, 1)</td>
<td>N(10, 3)</td>
<td>N(30, 7)</td>
</tr>
<tr>
<td>41, 42</td>
<td>BOS</td>
<td>JFK</td>
<td>N(3, 1)</td>
<td>N(9, 3)</td>
<td>N(22, 4)</td>
</tr>
<tr>
<td>51, 52</td>
<td>IAD</td>
<td>BOS</td>
<td>N(3, 1)</td>
<td>N(9, 3)</td>
<td>N(30, 7)</td>
</tr>
<tr>
<td>61, 62</td>
<td>IAD</td>
<td>JFK</td>
<td>N(3, 1)</td>
<td>N(8, 2)</td>
<td>N(30, 7)</td>
</tr>
</tbody>
</table>

Table 4 FHA’s capacity control policy

<table>
<thead>
<tr>
<th>Product</th>
<th>Orig.</th>
<th>Dest.</th>
<th>Displacement Cost</th>
<th>Fares</th>
<th>Accept Reservation?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>11, 12</td>
<td>JFK</td>
<td>IAD</td>
<td>$\lambda_{JFK-IAD} = $40</td>
<td>$203</td>
<td>$63</td>
</tr>
<tr>
<td>21, 22</td>
<td>JFK</td>
<td>BOS</td>
<td>$\lambda_{JFK-IAD} + \lambda_{IAD-BOS} = $93</td>
<td>$303</td>
<td>$93</td>
</tr>
<tr>
<td>31, 32</td>
<td>BOS</td>
<td>IAD</td>
<td>$\lambda_{BOS-IAD} = $30</td>
<td>$204</td>
<td>$44</td>
</tr>
<tr>
<td>41, 42</td>
<td>BOS</td>
<td>JFK</td>
<td>$\lambda_{BOS-IAD} + \lambda_{IAD-JFK} = $94</td>
<td>$304</td>
<td>$94</td>
</tr>
<tr>
<td>51, 52</td>
<td>IAD</td>
<td>BOS</td>
<td>$\lambda_{IAD-BOS} = $53</td>
<td>$203</td>
<td>$53</td>
</tr>
<tr>
<td>61, 62</td>
<td>IAD</td>
<td>JFK</td>
<td>$\lambda_{IAD-JFK} = $64</td>
<td>$204</td>
<td>$64</td>
</tr>
</tbody>
</table>

8 This capacity might have been adjusted up from the actual physical plane capacity to account for no-shows and cancellations. See Section 4 on overbooking.
Figure 1  Illustration of customer segmentation

(a) Revenue with customer segmentation

(b) Revenue without customer segmentation

Figure 1  Illustration of customer segmentation
Figure 4  FHA’s flight network
Figure 3 Determining the optimal booking limit in Example 2

\[ \text{EMSR1}(C - S_2) \]

\[ p_2 = 10 \]

\[ C - S_2^* \]
Figure 2 Relationship between booking limits, $S_i$, and protection levels, $y_i, i = 1, \ldots, n$