Chapter 7  Rate of Return Analysis: Single Alternative

- **Introduction**
  - The rate of return (ROR) for an investment involving one single payment $P$ and returning $F$ after 1 year
    \[
    \text{ROR} = i^* = \frac{F}{P} - 1.
    \]
  - Alternatively, $i^*$, is given by the solution to the equation
    \[
    -P + \frac{F}{1+i^*} = 0 \iff PW(i) = 0.
    \]
  - With more sophisticated projects involving several payments and receipts, the same equation, $PW(i) = 0$, gives the ROR.
  - E.g., for a $1,000 investment that pays $300 per year over 5 years, the ROR is 15.2%, as shown in the graph below.
• Definition
  ➢ ROR is the rate of interest paid on the *unrecovered balance* of an investment, so that the final receipt brings the balance to exactly 0 with interest considerations.

• Fact
  ➢ ROR is *not* the interest rate earned on the original investment amount.\(^1\)

• Deciding on an alternative based on ROR
  ➢ If \(i^* \geq MARR\), accept alternative.
  ➢ If \(i^* < MARR\), reject alternative.

• Range for \(i^*\) and relation to PW
  ➢ \(-100\% < i^* \leq +\infty\).
  ➢ \(i^* \geq MARR \iff PW(MARR) \geq 0\).

• ROR calculation using a PW or AW equation
  ➢ Set \(PW = 0\) or \(AW = 0\), this leads to
    o \(-PW_D + PW_R = 0\), or
    o \(-AW_D + AW_R = 0\),
    where the subscripts “\(D\)” and “\(R\)” denote disbursements (costs) and receipts.
  ➢ This usually involves finding the root(s) of an \(n^{th}\) degree polynomial where \(n\) is the project life.

\(^1\) *Installment financing* is paying interest based on the loan principal (initial balance). The rate that lenders provide to promote such loans is not their true ROR. (It may put the borrower at a great disadvantage.)
➢ By hand, ROR is found by solving one of the above equations by trial and error.
➢ Using a computer package, such as Excel, ROR can be usually found easily.
➢ To solve the equation quickly, use a good starting solution.

• Starting ROR Solution

➢ A good starting value for ROR is found as follows:
   o “Convert” all disbursements into either a single or uniform amount without considering time value of money. (This is a rough approximation.)
   o “Convert” all receipts into either a single value or a uniform series.
   o Solve the resulting \( PW = 0 \) equation which will be of the form \( PW_D = PW_R \times Factor(i*) \)
➢ The starting value is an approximation to ROR.
➢ Another good way to get a starting solution, especially when using a computer, is to plot PW (or AW) versus \( i \).
➢ This allows visual identifying a range for values of \( i* \).

• Solution by computer (Excel)

➢ In Excel, if cash flows, over \( n \) years, involve an initial payment, \( P \), followed by a uniform series of receipts, \( A \), and then a final (salvage) value, \( F \).
➢ Then, ROR is given by the function \( RATE(n, A, P, F) \).
If the cash flows do not follow this particular pattern, an initial “guess” value should be determined.

Then, cash flows are to be inputted in detail (say in the range first_cell:last_cell).

Then, ROR is given by \( IRR(first\_cell:last\_cell, \text{guess}) \).

- **Advantages of using ROR**
  - No need to estimate MARR.
  - It is somewhat intuitive and easy to understand.
  - Investors like it.

- **Disadvantages of using ROR**
  - For some types of cash flows the ROR method can be computationally difficult.
  - Some cash flows will result in multiple \( i^{*} \) values. This raises questions as to which, if any, \( i^{*} \) value to use.
  - Special procedure is required when comparing multiple alternatives (Chapter 8).

- **Checking whether there are multiple ROR values**
  - There are two tests to check whether a cash flow series could have multiple ROR values.
  - The tests are based on the idea that the ROR is one of the roots of the polynomial

\[
PW(c) = \sum_{t=0}^{n} F_t c^t \text{, where } c = 1/(1+i)
\]

and \( F_t \) is the cash flow at time \( t \).
- Descartes’ rule of signs.
  The maximum number of roots of a cash flow series is equal to the number of sign changes of cash flows.

- Norstrom’s criterion.

  Let $S_t = \sum_{r=0}^{t} F_r$ be the cumulative cash flows at time $t$. The cash flow series has a unique ROR if:
  - $S_0 < 0$;
  - The series $S_0$, $S_1$, …, $S_n$, changes sign only once (from minus to plus).

  When the two tests indicate that multiple ROR values may exist the next step is to plot $PW(i)$ vs. $i$ to find out how many RORs really exist.

- Conventional / unconventional cash flow series
  - A cash flow series is said to be conventional if it changes sign only once (from minus to plus).
  - Otherwise, the series is called unconventional.
  - Fortunately, conventional cash flows admit a unique ROR (based on Descartes’ rule).
  - Unconventional cash flows may admit multiple RORs.
  - There is no universally accepted way for analyzing these cash flows.
  - Some multi-ROR analysis methods are based on assuming a reinvestment rate for positive cash flows.
• **Example of unconventional cash flows with multiple RORs**
  - In Example 7.4 (text), there are two ROR for the cash flow series, in $K, (2, -0.5, 8.1, 6.8).
  - The two values are 7.47% and 41.35% (found using the Excel IRR() function.)
  - Here it is not obvious which value is the right ROR.