Chapter 12 Selection from independent projects under budget limitation

- Basic problem (capital budgeting problem)
  - An investor can choose from a set $S$ of $n$ independent projects.
  - At year 0, the investor has only $b$ dollars to invest.
  - Suppose that each project $j \in S$ requires an initial investment of $NCF_{j0}$.
  - Let $NCF_{jt}$ be the net cash flows of project $j$ at year $t$.
  - Suppose that the investor can provide any capital needed beyond year 0.
  - The problem is to select the subset of projects, $A \subset S$ that maximizes the investor’s fortune while meeting the budget constraint at year 0.

- Solution approach
  1. Identify “feasible bundles” that satisfy the budget constraint.
     That is, find all $A \subset S$, such that $\sum_{j \in A} NCF_{j0} \leq b$.
  2. Find the PW of each of these “feasible” bundles. The PW of a bundle is the sum of PWs of projects within the bundle.
     That is, $PW_A = \sum_{j \in A} PW_j$. 

3. Select the feasible bundle that gives maximum PW. That is, select \( A^* \) such that \( PW_{A^*} = \max_{A \in S} PW_A \).

- **Underlying assumption**
  - PW of each project is determined on its own life span. No LCM or equal life approach is needed.
  - This is equivalent to assuming that positive cash flows of short life projects in a bundle are reinvested throughout the life of the longest life project at the MARR. (See pages 430-432 of text.)

- **Solving the capital budgeting problem using ILP**
  - Solving the capital budgeting problem by hand can be tedious especially when the choice set \( S \) has many projects (\( 2^n \) bundles should be considered).
  - Integer linear programming (ILP) is an optimization technique that can be used to solve this problem quickly using a computer.
  - In ILP, each project \( j \in S \) is assigned a binary variable \( x_j \). The variable \( x_j \) can take on two values 0 and 1.
  - If \( x_j = 0 \), the project is not selected. If \( x_j = 1 \), the project is selected.
The ILP “model” for the capital budgeting problem is as follows:

\[
\begin{align*}
\text{max} & \quad Z = \sum_{j \in S} PW_j x_j \\
\text{subject to} & \quad \sum_{j \in S} NCF_{j0} x_j \leq b.
\end{align*}
\]

The ILP problem can be solved using several commercially available software (e.g., Microsoft Excel Solver).

The solution to the ILP is denoted by \(x^* = (x_1^*, x_2^*, \ldots, x_n^*)\).

A project \(j\) with \(x_j^* = 1\) is selected. Otherwise, \(j\) is not selected.

### Approximate (Heuristic) Capital Budgeting Solutions

Approximate solutions to the capital budgeting ILP is obtained by ranking projects according to benefit-cost ratio.

This approximation method assumes that each project involves an initial outlay of cash (first cost) followed by a series of benefits.

The benefit-cost ratio for each project is the ratio of the present value of benefits to the initial cost.

The approximate solution is obtained by selecting the project with the highest benefit-cost ratio, then the project with the second highest ratio, and so on, until the budget is exhausted.